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Pension Fund Investment Management: An Analysis of the Funding Ratio

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Abstract

We study the behavior of the funding ratio of a defined benefit pension plan by considering the development of both the asset portfolio and the liabilities. We also consider the real funding ratio by fully indexing the liabilities to inflation. The components that make up the assets and the liabilities are modelled by (1) a vector autoregression, (2) separate univariate regressions and (3) AR(1) models, respectively. AR(1) model predicts the lowest nominal and real funding probability due to its high prediction on the mean and variance of funding ratio. The vector autoregression, capturing mean reversion in stock returns, provides the lower long-term nominal underfunding probability among the three models. Parameter robustness checks in the VAR model shows both the nominal and the real underfunding probability are sensitive to parameter uncertainty.

1 Introduction

In the beginning of this century, the pension fund industry has gone through a double whammy of decreasing asset returns and increasing liabilities. This caused substantial underfunding and as a consequence regulatory measures were sharpened. As a pension fund's objective is to meet (future) pension obligations, the funding ratio has become a crucial criterion to assess the financial health of a pension fund by regulators. The funding ratio is defined as the market value of the assets divided by the market value of the liabilities. To meet a certain target of the funding ratio, partial focus on asset returns or liabilities alone will only result in vain, while emphasis on the interaction between assets and liabilities is the proper approach to gain insight on the development of funding ratio.

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Asset returns are influenced by macroeconomic factors such as interest rate and inflation, while we also know that the liabilities are sensitive to the similar factors. We suspect that the volatility caused by the same or related factors to both assets and liabilities may cancel out, and thus make the funding ratio predictable or drive it in a certain pattern. To the best of our knowledge, there have been no paper exclusively and explicitly studying the funding ratio. Therefore this paper comes to fill this gap by exclusively focusing on the funding ratio and using the probability of underfunding to explain its behavior. We choose this focus because it is practically desirable as pension fund regulators impose rules (such as nFTK in the Netherlands) on both the funding ratio and the probability of underfunding. In addition, the contribution and indexation policy which are more concerned by pension plan sponsors and participants are very much dependent and contingent on the funding ratio.

Considering both assets and liabilities in studying the pension funds is not novel. In Leibowitz (1994), "funding ratio return" is coined to analyze the asset allocation of a pension fund. Instead of assets-only return in traditional sense, liabilities are also included as a negative return to the asset portfolio choice. Hoevenaars, Molenaar, Schotman & Steenkamp (2005) extend this approach by adding more alternative assets and assume a constant maturity bond as a proxy for pension fund liability. But they model the liabilities separately from the modeling of the assets, this will result in the ignorance of potential interaction between the return dynamics of the liabilities and the assets. Binsbergen & Brandt (2005) do consider such interaction when modeling the funding ratio. But since their research interest is not per se on the funding ratio, they use a very reduced form of VAR, namely only long term interest rate and short term interest rate to account for the change of value in liabilities. Such simplification may not fully capture the interaction between the assets and liabilities. Our paper will follow the assumption of previous studies to use a constant maturity bond as a proxy for pension liability, but we will model its return in the same system as the asset returns so to capture the potential dynamics between the market value of assets and liabilities.

In addition, in the empirical finance literature a large amount of papers report that asset returns are to some extent predictable[1]. This predictability changes the long term risk-return profiles of important asset classes like stocks. Stock returns show mean reversion over long time horizon. In light of such predictability studies, we would like to explore their impact on the funding ratio and in particular, the underfunding probability. Our results show that mean reversion induce faster declining probability of underfunding than the model which does not capture mean reversion.

We also test the impact of model uncertainty and parameter uncertainty of return dynamics on the underfunding probabilities. Asset returns can be modeled by Vector AutoRegression (VAR) models as in Campbell & Viceira (2005), and can also be described by univariate models like AR(1). Our results shows that the AR(1) model provides the lowest underfunding probabilities than other models we used. We also examine the impact of small changes in the parameters in the VAR model on underfunding probabilities. We find that this simple way of dealing with the consequences of parameter uncertainty has considerate influence on the nominal underfunding probabilities, but less influence on the real underfunding probability.

The rest of the paper is structured as follows. In Section 2 we model pension funds' funding ratios as a function of a relevant group of variables. In Section 3 we describe three different models for asset returns. After introducing data, estimation and simulation methods for asset returns in Section 4, we analyze the results in Section 5. Section 6 concludes.

2 Model pension funds

This section describes our modeling of pension funds' funding ratio determined by assets and liabilities. Besides asset returns, contribution and indexation policies also play a role in determining the value of the funding ratio. However, we want to focus on the impact from investment policy, so we make the following assumptions as in Binsbergen & Brandt (2005) and Hoevenaars et al. (2005):

[1]For example Keim & Stambaugh (1986), Campbell (1987), Campbell & Shiller (1988a), Fama & French (1988), and Campbell (1991).

a. The fund is in a stationary state, i.e. the distribution of the age cohorts and their respective nominal pension rights are constant over time. So the market value of the liabilities of a fund only vary with the interest rate and inflation rate.
b. Each year inflows from contributions equal outflows for benefit payment. So we can model that the asset value is only determined by the investment results.

2.1 Nominal funding ratio

This part is to discuss the funding ratio when a fund promises only nominal pension benefit at its maturity. Suppose at time t, this fund's assets is A_t and invested in stocks and bonds with a weight vector λ, which is fixed for all times. After one period, the value of assets has increased to $A_{t+1} = A_t(1 + x_{t+1}^T \lambda)$, where $x_{t+1} = (x_{s,t+1} x_{b,t+1})'$, and$x_{s,t+1}$ and $x_{b,t+1}$ are the respective stock and bond return during period t and $t + 1$.

Based on the assumptions, the pension fund liability can be represented by an m-year constant maturity bond. m can be the average duration of a pension fund liability. Assuming discounted liability at time t is predetermined at L_t. So at time $t+1$, the market value of the fund liability is $L_{t+1} = L_t(1 + r_{t+1})^2$, where r_{t+1} is the holding period return (bond return in the later text) of the m-year constant maturity bond during period t and $t + 1$. Based on such definition we can derive the relationship between two consecutive funding ratios as

$$F_{t+k} = F_{t+k-1} \frac{1 + x_{t+k}^T \lambda}{1 + r_{t+k}} \tag{1}$$

where F_{t+k} is the nominal funding ratio at time $t + k$, x_{t+k} and r_{t+k} are the gross return of the asset portfolio and the bond respectively. The formula shows that the change of the funding ratio in the next period is determined by the gross portfolio return relative to the gross return of the constant maturity bond within that period.

[2] We assume a constant liability at maturity is L, just like the face value of the bond. Then its present value at time t is $L_t = L * (1 + y_{t,m})^{-m}$, where $y_{t,m}$ is the yield to maturity of a m-year constant maturity bond at time t. Accordingly at time $t + 1$, $L_{t+1} = L * (1 + y_{t+1,m})^{-m} = L_t(\frac{1+y_{t,m}}{1+y_{t+1,m}})^m$. While the holding period return according to Campbell & MacKinlay (1997) is defined as $(\frac{1+y_{t,m}}{1+y_{t+1,m}})^m - 1$, so we can write down $L_{t+1} = L_t(1 + r_{t+1})$.

The funding ratio in multiple periods as a function of the initial funding ratio can be derived as

$$F_{t+k} = F_t \prod_{i=1}^{k} (\frac{1 + x_{t+i}^T \lambda}{1 + r_{t+i}}) \tag{2}$$

This formula shows that long-term funding ratio is dependent on the initial funding ratio and the ratio of accumulative asset return to accumulative bond return. This already has some implication for the funding ratio development as to be explained later.

2.2 Real funding ratio

The ambition of a pension fund is to provide inflation indexed or even wage-indexed pension benefit. This section considers a pension fund that targets the full inflation-indexed pension benefit and we model the so-called real funding ratio in the following.

The asset value is the same as in the nominal case that they are A_t and $A_{t+1}(= A_t(1 + x_{t+1}^T \lambda))$ at time t and $t+1$ respectively.

As for the real liability, its present value should be adjusted for the inflation rate. For a pre-determined value of real liability L_t^{real} at time t, the value of real liability at $t+1$ increases by the bond return and the inflation, so $L_{t+1}^{real} = L_t^{real}(1 + \pi_{t+1})(1 + r_{t+1})$, where r_{t+1} is the nominal bond return of the m-year constant maturity bond and π_{t+1} is the inflation rate during the period t and $t+1$.

Based on such definition, then the relationship between two consecutive real funding ratios is

$$RF_{t+k} = RF_{t+k-1} \frac{(1 + x_{t+k}^T \lambda)}{(1 + r_{t+k})(1 + \pi_{t+k})} \tag{3}$$

The above shows that relative to the current real funding ratio, the real funding ratio in the next period is determined by the gross portfolio return in the next period relative to the inflation-added bond return.

The real funding ratio in multiple periods as a function of the initial funding ratio is

$$RF_{t+k} = RF_t \prod_{i=1}^{k} \frac{1 + x_{t+i}^T \lambda}{(1 + r_{t+i})(1 + \pi_{t+i})} \tag{4}$$

3 Modeling returns

The previous section shows that the development of the funding ratio is dependent on the asset portfolio return and the liability portfolio return, so this section will be describing three models that we employ for the return dynamics to forecast returns. The asset portfolio consists of stocks and bonds, and the liability portfolio consists of only bonds. Our choices of models will try to make the distinction among models capturing predictability, or interrelation or none of them so to show the difference from their respective impact.

3.1 Multivariate model – VAR

Vector Auto Regression has been used extensively to describe asset returns as recently as in Campbell & Viceira (2002), Campbell & Viceira (2005) and Hoevenaars et al. (2005).

As in Campbell & Viceira (2005), we describe the stock and bond returns by using their lagged values and the lagged values of other state variables such as real interest rate, nominal interest rate, dividend yield and yield spread[3], which has been acknowledged as predicting variables. We choose these variables also because they capture the common factors such as interest rate and inflation rate that influence both the asset and the liability side of a pension fund. Specifically, let $z_t = (x_t, s_t)'$ where x_t is a vector of the asset returns we want to describe, i.e. stock return $x_{s,t}$ and bond return $x_{b,t}$. s_t is a vector of the state variables which are reported to have prediction power and include real interest rate, nominal interest rate, dividend yield and yield spread. The VAR model is defined as

$$z_{t+1} = A + Bz_t + \epsilon_{t+1}$$

[3] To mention a few, Campbell (1987) pointed out term structure of short-term interest rate can predict stock and bond return; Fama & French (1988) studied the dividend yield for stock return; Campbell & Shiller (1991) pointed out yield spread can forecast interest rate.

where A is a 6×1 constant vector, B is a 6×6 coefficient matrix for z_t, and $\epsilon_{t+1} \sim N(\mathbf{0}, \Omega)$.

3.2 Alternative models

There are other models that pension fund managers might consider to describe asset returns. Here we also adopt them to see what implications they have on the funding ratio and the underfunding probability.

First to consider is a series of unrelated univariate regressions similar to VAR, and we call them separate regressions (Sep), because they do not account for the covariance among the errors of each regression. This model does not capture the mean reversion of the long term stock return, but they do consider the interrelation between stocks and bonds. They are estimated as follows:

$$z_{t+1}^d = A^d + B^d z_t^d + \epsilon_{t+1}^d$$

where A^d is a 6×1 constant vector, B^d is a 6×6 coefficient matrix for z_t^d, and $\epsilon_{t+1}^d \sim N(\mathbf{0}, \mathbf{D})$. D is a diagonal matrix.

Second to consider is the simple AR(1) model,where each variable is only explained by its own lagged value. This means no interrelation between stocks and bonds return is considered. They are estimated as follows:

$$z_{j,t+1} = a_j + b_j z_{j,t} + \epsilon_{j,t+1}$$

where a_j is the constant and b_j is the coefficient for the variable j equation, $j = \{stock, bond, real\ rate, nominal\ rate, dividend\ yield, yield\ spread\}$, and $\epsilon_{j,t+1} \sim N(0, \sigma_j)$.

4 Data, estimation and simulation methods

4.1 Data description

We use quarterly data from the Center for Research and Security Prices (CRSP) of the University of Chicago for the period 1952Q2 to 2004Q4.[4]

Our modeling of the funding ratio requires the input of simple net return of assets, so we use the data of simple net returns rather than log and excess returns.[5]

To study the impact of the return models on the funding ratio, we adopt a stylized strategic asset mix of 60% stocks and 40% bonds as our pension fund portfolio.The return on stocks (including dividends) and the dividend yield are for a portfolio that includes all stocks traded on the NYSE, NASDAQ, and AMEX. Dividends are twelve-month moving sums of dividends paid on the just mentioned portfolio that includes all stocks traded on the NYSE, NASDAQ, and AMEX. The dividend yield is the ratio between the dividends and the prices. The return on bonds is represented by the return to a 10-year constant maturity T-bond.[6]

The pension fund liability is represented by 10-year constant maturity T-bond[7]. Its quarterly return can be obtained according to the loglinear relationship between log return and log yield as described by Campbell & MacKinlay (1997) that $ln(1 + r_{n,t+1}) \approx$

[4]The starting date 1952Q2 comes after the Fed-Treasury Accord that allowed short-term nominal interest rates to freely fluctuate and constant bond yield is available from Feds website.

[5]Simple net return is defined as $(P_{t+1} - P_t)/P_t$. We do not use log return because we do not need the convenience of calculating multiple period return and it also avoid the problem experienced by log return when calculating portfolio return from individual asset returns. We do not use excess return because in this way we do not have to add in the risk free rate separately in calculating the portfolio return. A similar example of using gross return can be found in Binsbergen & Brandt (2005) who use log of annual gross return to model stock return in their study of asset liability management.

[6]In studying the long-term portfolio choice, Campbell & Viceira (2005) use 5-year constant maturity T bond, while Budek, Schotman & Rolf (2006) use 10-year constant maturity T bond to represent long-horizon investment. In studying the pension fund portfolio choice, Hoevenaars et al. (2005) use a 10-year constant maturity T bonds to represent bond in pension fund portfolio. Nijman & Swinkels (2003) use Ibbotson long-term government bond index for bond investment when studying the commodity in pension funds.

[7]Binsbergen & Brandt (2005) proxy liability with a 15-year constant maturity bond. Hoevenaars et al. (2005) proxy liability with a 17-year constant maturity index-linked bond. Nijman & Swinkels (2003) use a 10 year T bond yield for liability return. Here we use 10-year bond instead of 15- or 17-year bond, as we believe the dynamics of both yields are similar which has no material influence on our results.

$D_n ln(1 + Y_{n,t}) - (D_n - \frac{1}{4}) ln(1 + Y_{n-1,t+1})$ where D_n is Macaulay's duration, calculated as $(1-(1+Y_{n,t})^{-n})/(1-(1+Y_{n,t})^{-1})$. $Y_{n,t}$ is the annualized yield of a n-year constant maturity bond at time t and n is 10 here. Such yield data is directly obtainable from US Federal Reserve Bank. The derived $r_{n,t+1}$ is the simple quarterly return of a n-year constant maturity bond at time t and as input for the bond return and the growth rate of liability.

The real interest rate is the ex-post real return on 90-day T-bills (i.e. the difference between the yield on T-bills and the inflation rate). We use return on the 90-day T-bill as our measure of the short term nominal interest rates. Yield spread is the difference between the yield on a zero coupon 10-year T-bond and the return on a 90-day T-bill.

4.2 Estimation results

We firstly give a statistical description of all six variables in our sample as shown in Table 1.

The estimates for VAR model using quarterly data 1952Q2:2004Q4 are reported in Table 2 and 3. The maximum eigenvalue of the coefficient matrix is 0.958 which indicates that the estimated VAR model is stationary. The second row in Table 2 corresponds to the equation for stock return. Dividend yield can predict stock return, and together with the other variables they can only explain 6.5% of the variation in stock return. The third row corresponds to the equation for bond return, which shows that the lagged stock return, nominal interest rate and yield spread can predict bond return.

Table 3 reports the covariance among shocks to each variable. Specifically, together with Table 2 it indicates the mean reversion of stock return. This behavior is induced by its predictability from dividend yield. Negative covariance (-0.000198) between the shocks to stock return and dividend yield means that a negative shock to dividend yield is accompanied by a positive contemporaneous shock to stock. But a low dividend also predicts a lower expected stock return due to positive coefficient (1.628). Figure 1 shows that the annualized standard deviation of stock return declined from 16% of one quarter horizon to about 8% in 50 year horizon.

As bond return is regarded, stock return induces mean reversion due to significant negative coefficient (-0.072) and positive shock correlation(0.00141). But the nominal interest rate and yield spread pull bond return in another direction. The total effect is a slight mean reversion displayed by the 10-year constant maturity bond return line in Figure 1. Together with Table 1, the figure indicates that for long term investment stocks and constant maturity bonds are better investments than short-term T bills as stocks can provide very high mean return with considerately decreased long-term variance, and constant maturity bonds can provide a slightly higher mean return but involving a lower variance than short-term T bill.

We estimate 6 separate univariate regressions using the same regressors as in VAR. The results of coefficients and the variances of the error terms are the same as in VAR except that there are no covariances among the error terms as shown in Table 3, i.e. the off-diagonal entries are 0s.

We also estimate 6 simple AR(1) models for the 6 variables individually, where each variable is to be only explained by its own lagged value. The results are shown in Table 4. We see that bond return, nominal interest rate and dividend yield can be very well described by their own lagged values.

4.3 Simulation method

Given the assumption of multivariate normal distribution of the error terms with the zero mean vector and the estimates of the covariance matrix, we are able to generate error terms for the future 200 periods (50 years), so that we can calculate the forecasted asset returns in the next quarter up till the next 50 years from the following:

$$z_{T+k} = \hat{A} + \hat{B} z_{T+k-1} + \epsilon_{T+k}$$

where z_T is the last observation in our sample, $k = 1 : 200$, $\hat{A}$ and $\hat{B}$ are the constant and coefficient estimates from the respective models.

Accordingly, with Equation (1) and (3) we can calculate the forecasted nominal and real funding ratio (F_{T+k} and RF_{T+k}, $k = 1 : 200$) in the next quarter up till the next 50 years.

We repeat the above return generation and funding ratio calculation for 1000 times. For each period k we are able to compute the probability of underfunding (i.e. at each point in time $T + k$, underfunding probability=(number of times when F_{T+k} or RF_{T+k} is bigger than 1)/1000). Therefore we are able to draw a line to show the underfunding probability over different time horizon .

Different return model prescribes a different pattern for underfunding probability. Figure 3 displays the nominal underfunding probability predicted by the three return models, while Figure 5 displays the real underfunding probability predicted by the three return models.

5 Analysis of funding ratio and probability of underfunding

Given the estimates of the asset return models, we can predict how the assets and liabilities of our pension fund evolve over time. Thereby we can predict the future funding ratio over different time horizon and compute the related probability of underfunding. The following three subsections analyze the results obtained in Section 4.3, namely a declining trend of nominal underfunding probability over longer horizon, the impact on the underfunding probability of alternative return models, and the impact of parameter uncertainty. We hope to give some insight on the development of the funding ratio of a given pension fund portfolio under a passive investment policy.

5.1 Feature of the long-term funding ratio

Firstly, figure 2 shows the mean and one standard deviation of the funding ratio over different time horizon. We observe an increasing mean funding ratio over time irrespective of the return models. Such trend can be explained by our modeling of the funding ratio, where the asset portfolio consists of stocks and bonds and the liability portfolio is represented by only the bonds. Assuming the initial nominal funding ratio is 1, Equation (1) and (2) indicates that the state of overfunding or underfunding is just contingent on the returns earned by the asset portfolio relative to that earned by the liability portfolio, and in the long run it is contingent on the cumulative returns of the respective portfolios. Due to the distinctive risk-return profile of stocks and bonds, stocks earn a higher return than bonds in general. Thus over time mean funding ratio will increase.

Secondly, we compute the nominal underfunding probability from the simulation, and the result in Figure 3 shows that it declines over time irrespective of the models we use for return dynamics. Specifically, in 50 years time, VAR and AR (1) models have prescribed that the underfunding probability will decrease from 45% to less than 3%, while the separate regression model predicts a underfunding probability to be around 20%.

As regards to the real funding ratio when pension funds try to fully index their liabilities with inflation, Equation (3) and (4) shows that the evolvement of real funding ratio depends on the portfolio return relative to the inflation-enhanced bond return. So only when portfolio obtain a higher real return than nominal bond return, can the real funding ratio get higher. Simulation results in Figure 4 shows mean funding ratio decreases in VAR and Sep model, but increases in AR(1) model. Figure 5 show that in either short or long-run, probability of real underfunding increases in all three models. This reflects the fact that a passive investment portfolio of stocks and bonds can hardly meet the inflation-linked liabilities.

5.2 Model impact on funding ratio

Different models employed to describe returns delineate different pictures for the funding ratio and the underfunding probabilities.

Figure 3 shows that AR(1) model prescribes the fastest declining underfunding probability. Figure 2 gives more detailed information for this. AR(1) model prescribed the highest mean funding ratio, but also the highest variance. Simulation shows that its high mean value dominates the volatility and thus cause the underfunding probability to decline fast. Comparing VAR model and Sep model, even though VAR predicts a lower mean value of funding ratio, yet the funding ratio variance is lower, which has maintained the funding ratio above 1 continuously. Thus we see a lower underfunding probability than in Sep model. Such difference suggests that the mean value captured by VAR has contributed tremendously to improve the funding status.

In case of the real funding ratio in Figure 4, as in nominal case, AR(1) still predicts the highest mean return. Yet the high variance induces an increasing underfunding probability over time. In VAR and Sep models the simulation shows a declining mean funding ratio. This indicates that in VAR and Sep model the returns earned by the asset portfolio cannot beat the inflation-enhanced bond return earned by the liability portfolio. This is accompanied by a fast increasing underfunding probability.

Combining the simulation results in both nominal and real funding ratio, we find that VAR model predicts the lowest variance for both real and nominal funding ratio due to the fact that it captures the mean reversion of stock returns. AR(1) model predicts the highest mean real and nominal funding ratio due to the fact that it ignores the interrelation between stocks and bonds. Such ignorance has allowed to generate high stock returns. All in all they suggest a considerate influence from the choice of return model on the funding status over time.

5.3 Sensitivity to parameter uncertainty

Any model is subject to the issue of sensitivity, so we also test how funding ratio is sensitive to the change in parameter uncertainty. As VAR is the best among the three models in describing asset returns and also popular among practitioners, we will focus the sensitivity tests on VAR model.

We increase the estimate for constant parameter of stock equation (-0.024) upward by 10%, only about 10% of its standard deviation (0.021). In Figure 6, the strategic mix under the new estimate produces a much lower underfunding probability than the original level. Under the new parameter for the constant in VAR, even the most risky allocation (100% stocks) for asset portflio will predict a lower underfunding probability than the base case. This reflects that funding status is very sensitive to the parameter uncertainty, and the tool of changing portfolio choice to handle parameter uncertainty is not effective. In another direction, we decrease the constant parameter of the stock equation in VAR, and Figure 7 shows a considerate increase of underfunding probability under the new estimate. A substantial decrease of the holding of stock to about 10% can help to maintain the original underfunding probability. These two figures illustrate that the nominal underfunding probability is very sensitive to the parameter uncertainty either in short or long term horizon. Adjustment of asset allocation is a very limited remedy to maintain the nominal financial status when parameter uncertainty is considered.

We do the same sensitivity test for the real funding ratio. Figure 8 and Figure 9 demonstrate that when the constant of the stock equation is increased or decreased by 10%, the 60/40 portfolio produces also very different underfunding probability from the original level. An respective decrease of 10% in Figure 8 and an increase of 10% in Figure 9 in the stock allocations will recover the original probability of real underfunding in base case. It shows that real underfunding probability is also very sensitive to the parameter uncertainty, but by changing the allocation, such parameter uncertainty can be handled better than in the nominal case.

The sensitivity test shows that underfunding probability, real and nominal, are very sensitive to parameter uncertainty, and it confirms the findings in Hoevenaars, Molenaar, Schotman & Steenkamp (2006). In addition, it also implies the effectiveness of manager's ability to handle the uncertainty. When a fund manager according to his personal experience thinks stock return is overvalued, and in order to maintain a certain target of nominal underfunding probability, he has to decrease stock allocation. Our simulation shows this is not effective, because cutting down stock holding to as low as 10% will not help. While to maintain a certain target of real underfunding probability, he can reach the goal by adopting a relatively feasible measure such as increasing the stock allocation by 10%.

6 Conclusions

In this paper we study the development of the funding ratio over different time horizon by considering the market valuation of both the assets and the liabilities of a pension fund. We derive the funding ratios in nominal and real terms as a function of the initial funding ratio and the asset returns. We use vector autoregression, separate univariate regressions and AR(1) models, respectively, to describe the dynamics of asset returns and state variables. From these models we simulate future returns and compute future funding ratio over different time horizons. We analyze the evolvement of the funding ratio over different time horizons and the impact of the return models on the underfunding probabilities. We also use simulation methods to examine the impact of small changes in parameters.

When pension funds only care about the nominal obligations, their liabilities increase in value approximately by the rate of bond returns while the assets increase by the rate of portfolio returns. As a result over long term the funding ratio is bigger than the initial funding ratio. Further consideration of the mean reversion of stock return captured by the VAR model predicts that the probability of underfunding over the long term will be even lower. AR(1) model, which does not consider the interrelation between asset returns, predicts the lowest underfunding probability due to the high predicted mean funding ratio. The nominal funding status is very sensitive to parameter uncertainty.

When pension funds try to index the liabilities to inflation, real funding ratio deteriorates and underfunding probability increases over time. This shows that a passive investment portfolio won't assure a pension fund to realize full inflation indexation. The impact from the different return models still exists as in the real case. The real funding status is also very sensitive to parameter uncertainty, but they can be remedied partially with asset allocation.

As a general conclusion, we can relieve the worry about pension fund's ability to meet the final objective of paying the promised nominal pension benefit given that they start with a break-even situation and can manage the intertemporal cash inflows and outflows. Yet for the ambition of the full inflation-indexed liabilities, pension funds need to do something more than passive investment. In addition, it should be highly awared that the underfunding probability is very sensitive to the model and parameter uncertainty.

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Table 1: Descriptive statistics sample

Descriptive statistics for the entire sample of 211 observations(1952Q2-2004Q4). All returns are quarterly simple return. Variables are real return of 90-day Treasury bill, stock, 10 year constant maturity US government bond, nominal rate of 90-day Treasury bill, dividend yield and yield spread.

	Real t bill	Stock	Bond	Nominal t bill	Dividend yield	Yield spread
Mean	0.0035	0.0305	0.0153	0.0129	0.0329	1.3037
Median	0.0034	0.0375	0.0109	0.0126	0.0324	1.2750
Maximum	0.0287	0.2447	0.1898	0.0385	0.0577	4.0630
Minimum	-0.0156	-0.2454	-0.1132	0.0016	0.0112	-3.3800
Std. Dev.	0.007	0.083	0.039	0.007	0.011	1.255
Skewness	0.131	-0.519	0.685	1.033	-0.030	-0.235
Kurtosis	4.418	3.849	5.281	4.407	2.515	3.581

Table 2: Estimates for VAR

The table reports full sample OLS parameter estimates of VAR $z_{t+1} = A + Bz_t + \epsilon_{t+1}$,regressors are lag value of real Treasury bill rate, stock return, bond yield, nominal Treasury bill rate,dividend yield and yield spread. T ratios are in parentheses. The last column is the R^2 for the respective equations.

	Constant	Real t bill	Stock	Bond	Nominal t bill	Dividend yield	Yield spread	R squared
Real t bill	-0.001 (-0.50)	0.391 (5.55)	-0.002 (-0.42)	-0.012 (-0.98)	0.254 (3.67)	-0.021 (-0.51)	0.000 (1.30)	0.258
Stock	-0.024 (-1.11)	0.277 (0.30)	0.011 (0.16)	0.266 (1.62)	-0.967 (-1.06)	1.628 (2.96)	0.006 (1.20)	0.065
Bond	-0.007 (-0.72)	0.309 (0.72)	-0.072 (-2.21)	-0.107 (-1.41)	1.304 (3.08)	-0.027 (-0.11)	0.007 (3.05)	0.101
Nominal t bill	0.002 (4.41)	0.007 (0.32)	0.001 (0.58)	-0.037 (-9.88)	0.947 (45.09)	-0.006 (-0.46)	-0.001 (-5.19)	0.937
Dividend yield	0.001 (2.07)	-0.022 (-0.72)	-0.001 (-0.26)	-0.006 (-1.16)	0.045 (1.47)	0.947 (51.85)	0.000 (-1.34)	0.941
Yield spread	0.307 (1.49)	0.948 (0.11)	-0.953 (-1.40)	-0.629 (-0.40)	5.217 (0.59)	-2.253 (-0.43)	0.801 (16.80)	0.622

Table 3: Variance and covariance matrix of residuals

The table reports covariance matrix for the residuals of VAR $z_{t+1} = A + Bz_t + \epsilon_{t+1}$. Diagonal entries are variances; off-diagonal entries are covariance.

Equation for	Variance and covariance ($\times 10^{-4}$)					
Real t bill	0.38	-	-	-	-	-
Stock	1.18	65.65	-	-	-	-
Bond	0.89	5.95	14.1	-	-	-
Nominal t bill	-0.007	0.007	0.05	0.03	-	-
Dividend yield	-0.04	-1.98	-0.2	0.002	0.07	-
Term spread	6.9	40.1	66.03	1.56	-1.6	6114

Table 4: Estimates for simple AR (1) regression

This table reports the parameter estimates of $z_{j,t+1} = a_j + b_j z_{j,t} + \epsilon_{j,t+1}$. Each variable is only regressed on its own lagged value. The third and fourth columns report the variance of the residuals and the R^2 of the corresponding equation. T ratios are in parentheses.

Equation for	Coefficient of lagged value	Variance of residual ($\times 10^{-4}$)	R squared
Real t-bill rate	0.450 (7.33)	0.393	0.204
Stock return	0.040 (0.58)	68.11	0.002
Bond	-0.013 (-0.19)	15.24	0.000
Nominal t bill	0.938 (39.18)	0.064	0.880
Dividend yield	0.963 (56.59)	0.074	0.938
Yield spread	0.786 (18.41)	6020	0.617

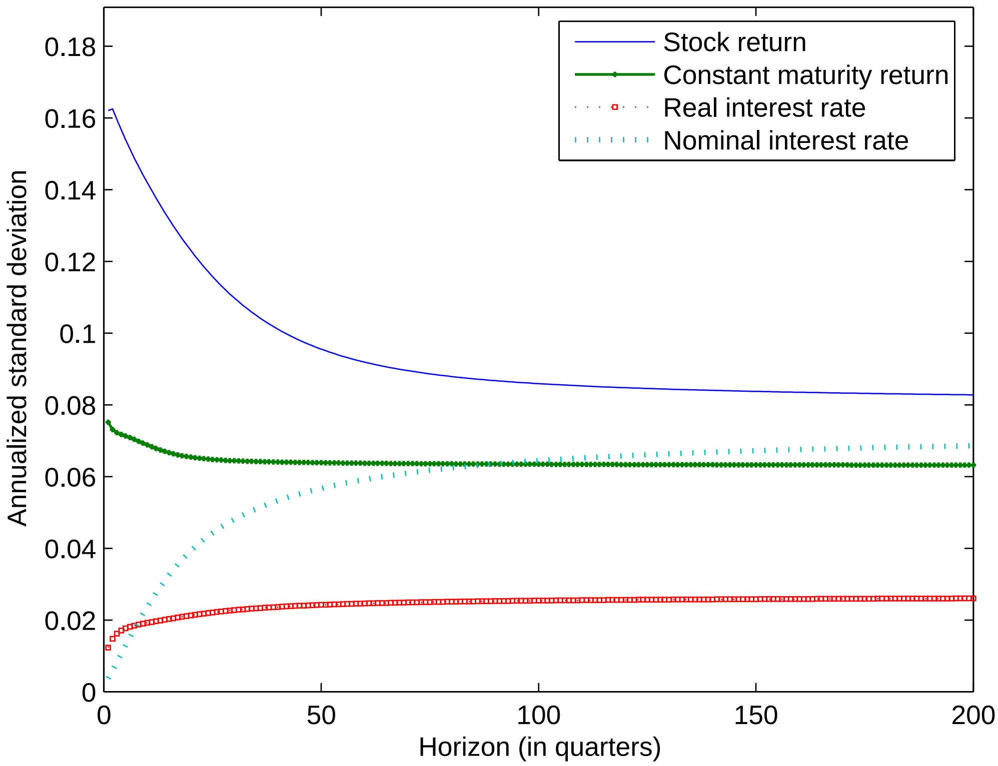

Figure 1: Annualized standard deviation
Annualized standard deviation over different time horizon for nominal stock return, nominal return of 10Y constant maturity bond, real and nominal 90D T bill rate implied by quarterly VAR estimates, 1952Q2-2004Q4.

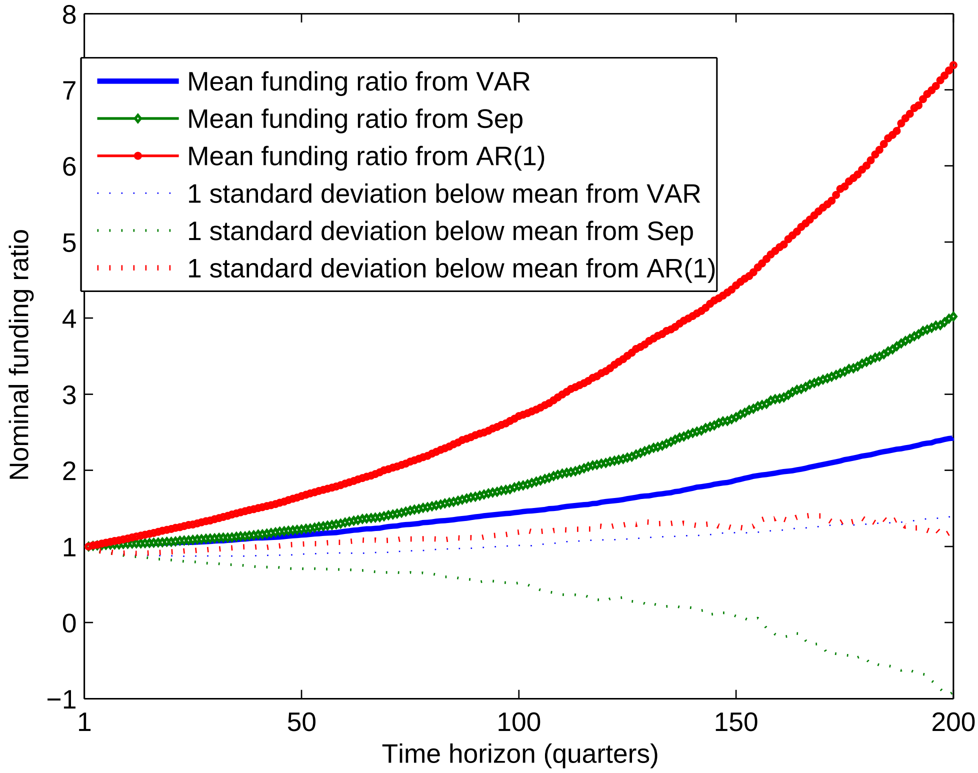

Figure 2: Distribution of simulated nominal funding ratio over time prescribed by different return models

Solid lines represent the mean, and dotted lines represent one standard deviation below the respective mean of the nominal funding ratio over time. Initial nominal funding ratio is 1 and fund portfolio is fixed at 60% stocks and 40% bonds.

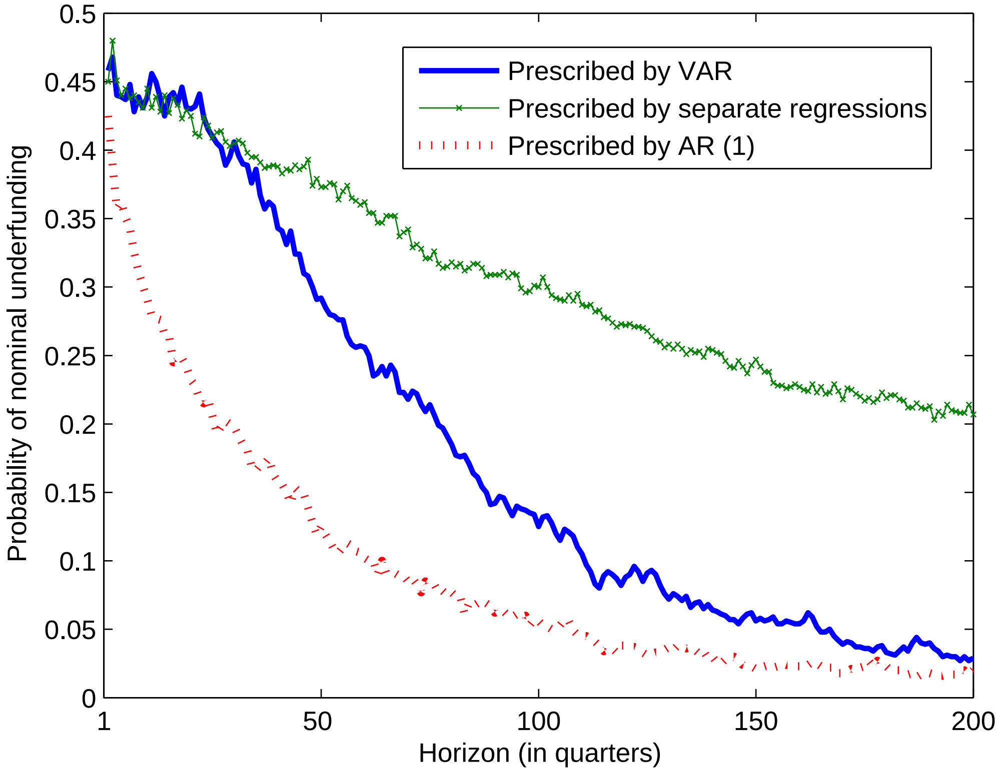

Figure 3: Model impact on nominal underfunding probability
Each line represents probability of underfunding in nominal term over time when return dynamics is described by the respective model, initial nominal funding ratio is 1 and fund portfolio is 60% stocks and 40% bonds.

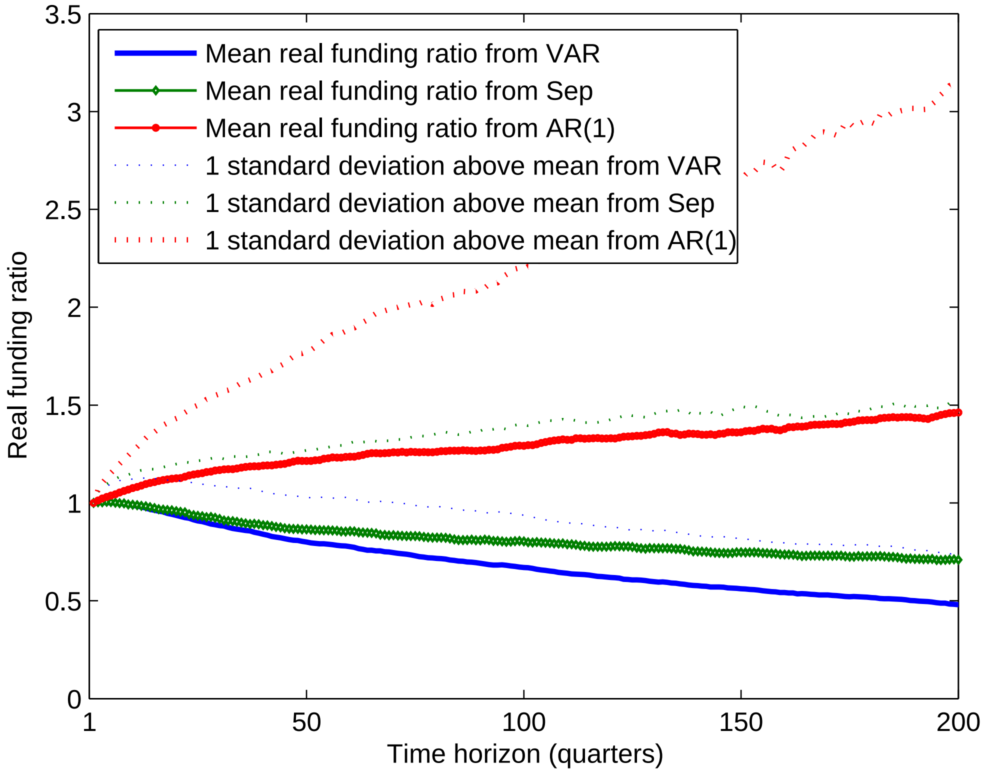

Figure 4: Distribution of simulated real funding ratio over time prescribed by different return models
Solid lines represent the mean, and dotted lines represent one standard deviation above the respective mean of the real funding ratio over time. Initial real funding ratio is 1 and fund portfolio is fixed at 60% stocks and 40% bonds.

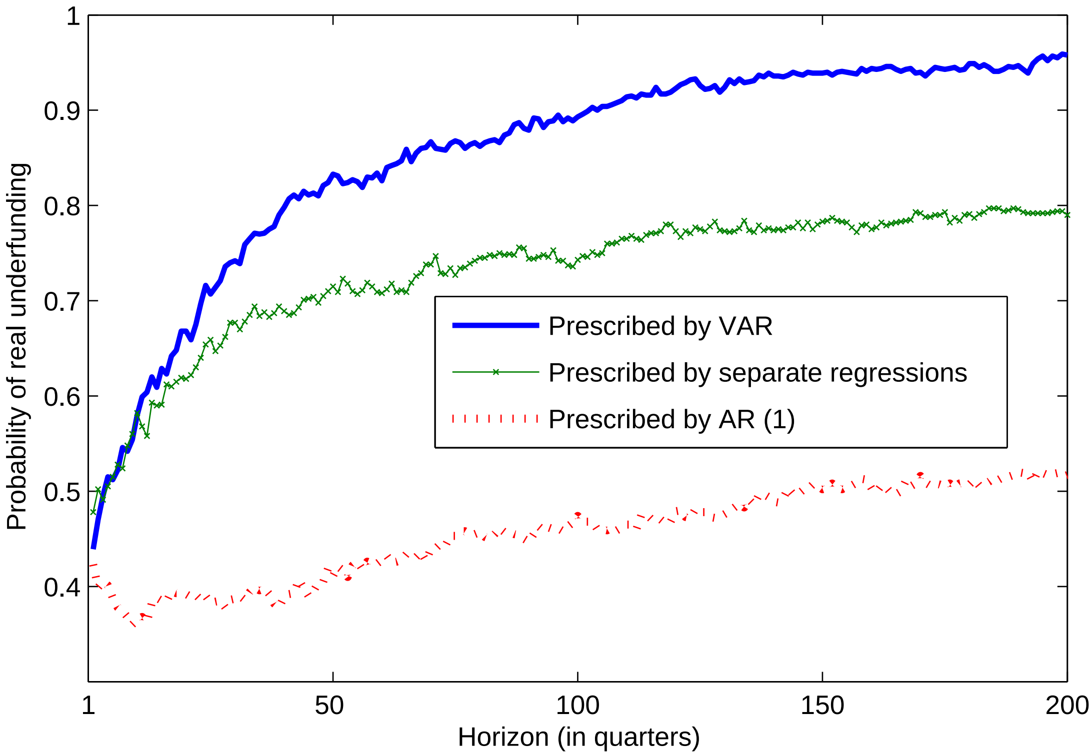

Figure 5: Model impact on real underfunding probability
Each line represents probability of underfunding in real term over time when return dynamics is described by the respective model. Initial real funding ratio is 1 and fund portfolio is 60% stocks and 40% bonds.

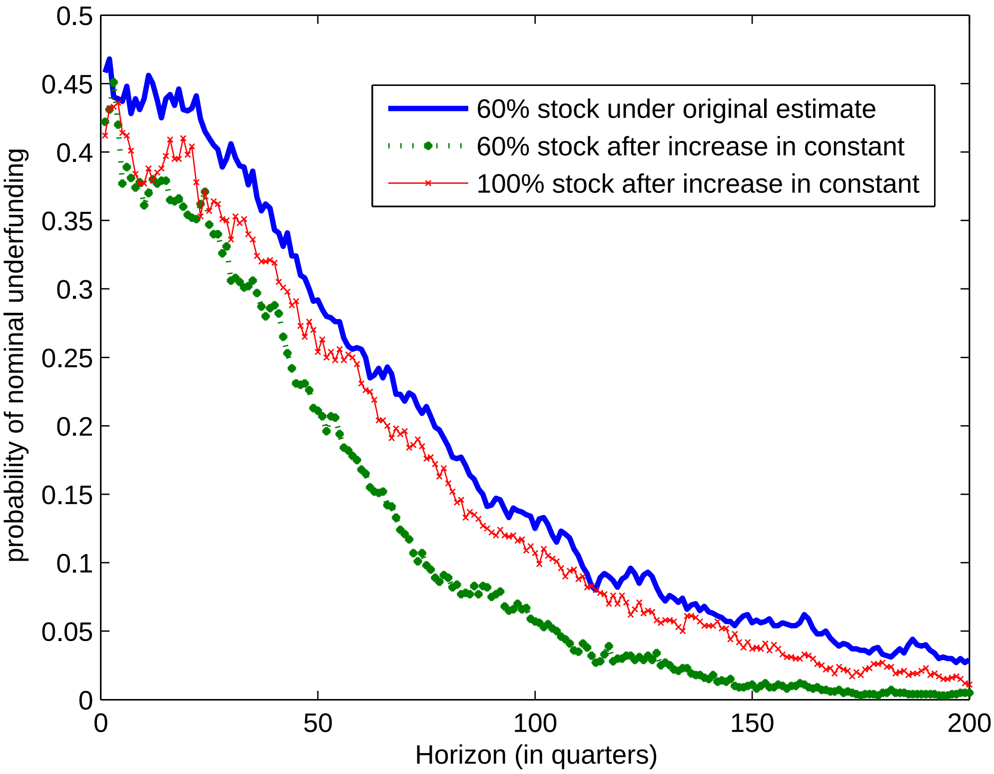

Figure 6: Nominal funding ratio's sensitivity to parameter uncertainty
Solid line is nominal underfunding probability by using original estimate in VAR of a 60/40 pension portfolio. Dot line represents nominal underfunding probability when constant parameter of stock equation in VAR increases by 10%. Marked line represents nominal underfunding probability given by a portfolio composition which can attain the underfunding probability closest to the original level.

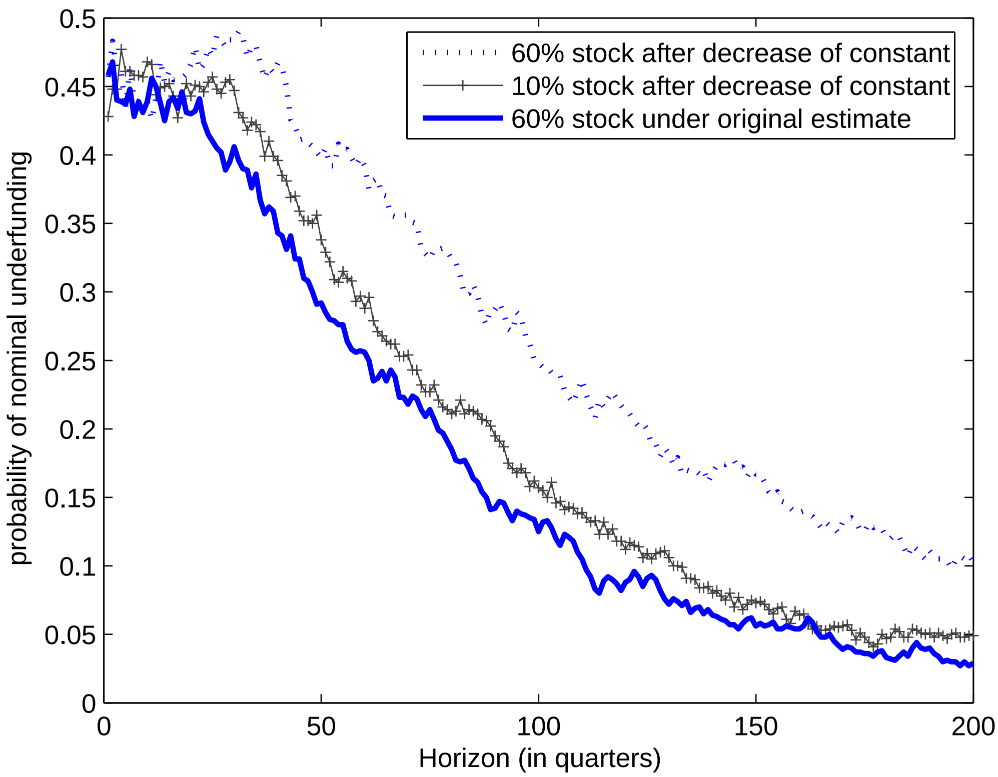

Figure 7: Nominal funding ratio's sensitivity to parameter uncertainty
Solid line is nominal underfunding probability by using original estimate in VAR. Dot line represents nominal underfunding probability when constant parameter of stock equation in VAR decreases by 10%. Marked line represents nominal underfunding probability given by a portfolio composition which can attain the underfunding probability closest to the original level.

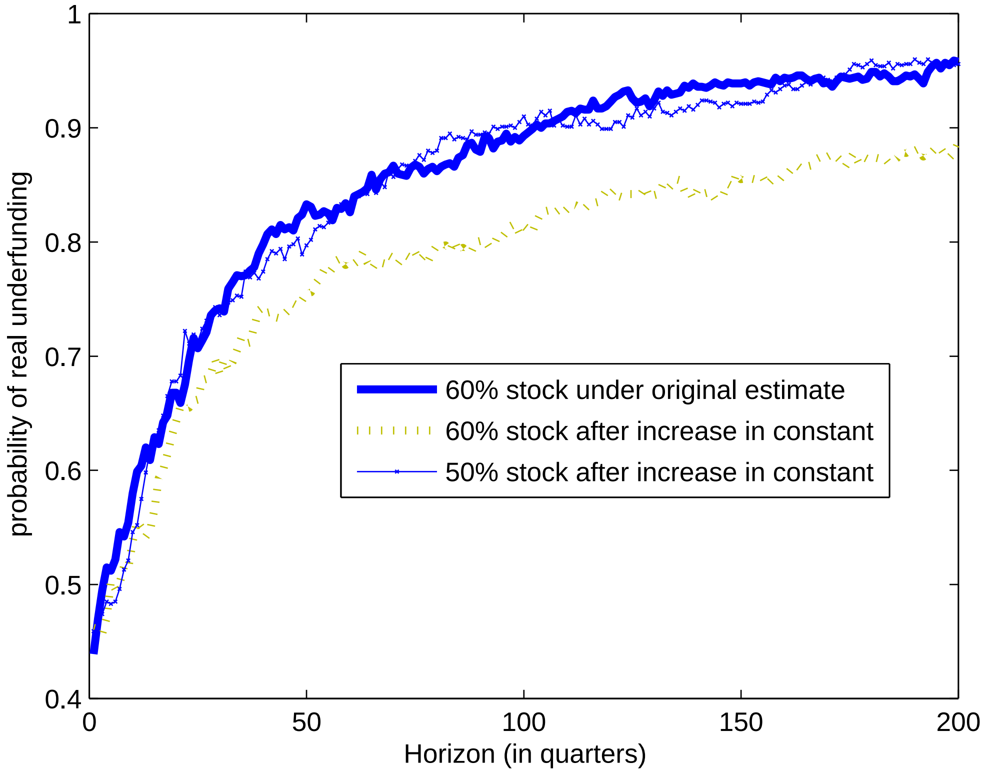

Figure 8: Real funding ratio's sensitivity to parameter uncertainty
Solid line is real underfunding probability by using original estimate in VAR. Dot line represents real underfunding probability when constant parameter of stock equation in VAR increases by 10%.Marked line represents real underfunding probability given by a portfolio composition which can attain the underfunding probability closest to the original level.

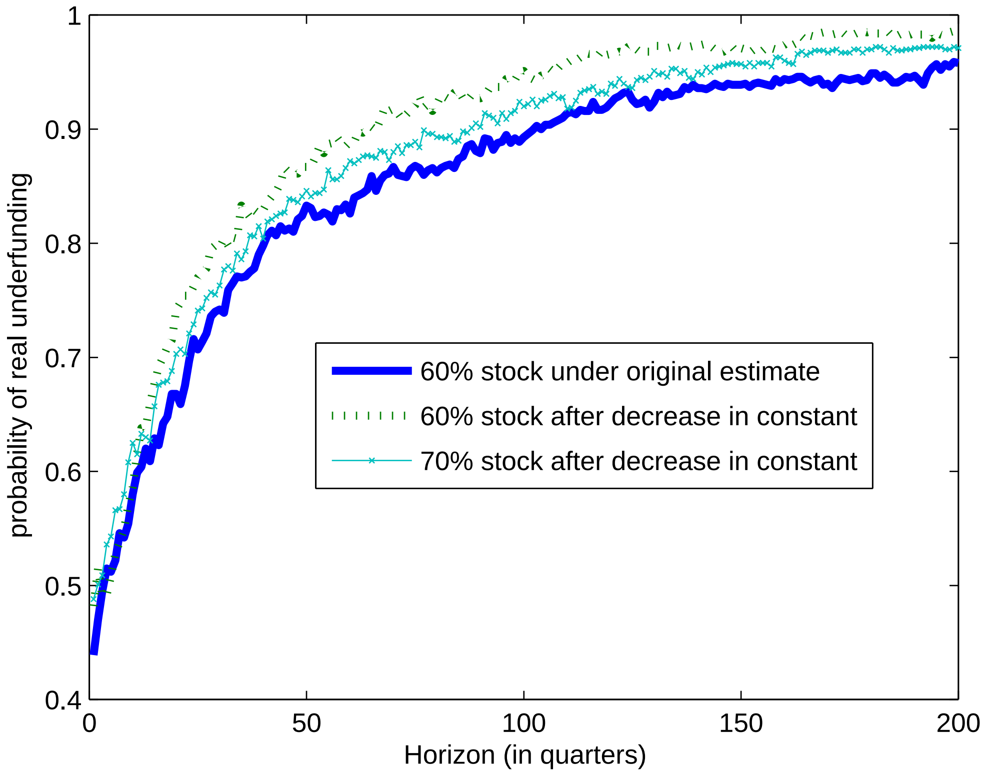

Figure 9: Real funding ratio's sensitivity to parameter uncertainty
Solid line is real underfunding probability by using original estimate in VAR. Dot line represents real underfunding probability of the original portfolio when constant parameter of stock equation in VAR decreases by 10%. Marked line represents real underfunding probability given by a portfolio composition which can attain the underfunding probability closest to the original level.